Questions

Q1.

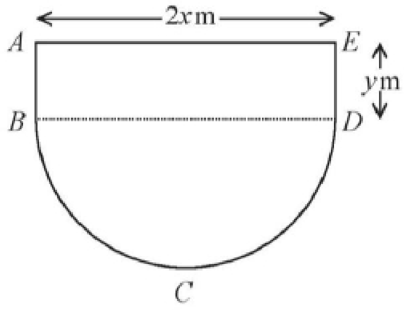

Figure 4

Figure 4 shows the plan view of the design for a swimming pool.

The shape of this pool *ABCDEA* consists of a rectangular section *ABDE* joined to a semicircular section *BCD* as shown in Figure 4.

Given that *AE* = 2*x* metres, *ED* = *y* metres and the area of the pool is 250 m^2,

(a) show that the perimeter, *P* metres, of the pool is given by

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}$$

(4)

(b) Explain why $0 < x < \sqrt{\frac{500}{\pi}}$

(2)

(c) Find the minimum perimeter of the pool, giving your answer to 3 significant figures.

(4)

(Total for question = 10 marks)

Q2.

A lorry is driven between London and Newcastle.

In a simple model, the cost of the journey £C when the lorry is driven at a steady speed of v kilometres per hour is

$$C = \frac{1500}{v} + \frac{2v}{11} + 60$$

(a) Find, according to this model,

(i) the value of v that minimises the cost of the journey,
(ii) the minimum cost of the journey.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

(b) Prove by using $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$ that the cost is minimised at the speed found in (a)(i).

(2)

(c) State one limitation of this model.

(1)

(Total for question = 9 marks)

Q3.

A curve has equation $y = \text{g}(x)$.

Given that

- $\text{g}(x)$ is a cubic expression in which the coefficient of x^3 is equal to the coefficient of x
- the curve with equation $y = \text{g}(x)$ passes through the origin
- the curve with equation $y = \text{g}(x)$ has a stationary point at (2, 9)

(a) find $\text{g}(x)$,

(7)

(b) prove that the stationary point at (2, 9) is a maximum.

(2)

(Total for question = 9 marks)

Q4.

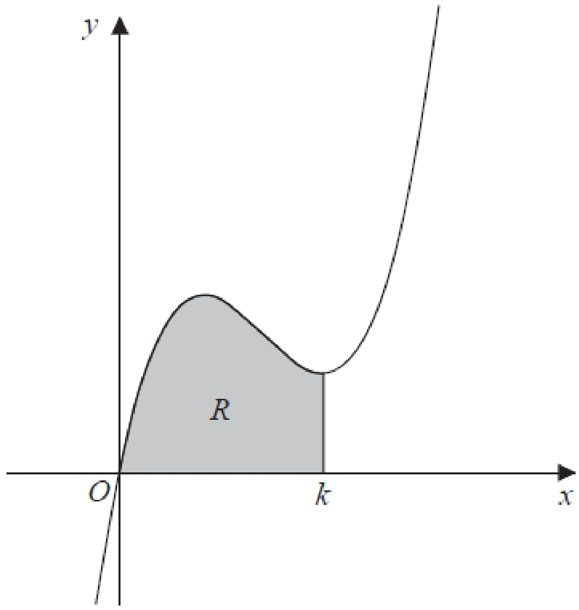

Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 2x^3 - 17x^2 + 40x$$

The curve has a minimum turning point at $x = k$.

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = k$.

Show that the area of R is $\frac{256}{3}$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(7)

(Total for question = 7 marks)

Q5.

A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius r cm and height h cm.

In the model they assume that the can is made from a metal of negligible thickness.

(a) Prove that the total surface area, S cm^2, of the can is given by

$$S = 2\pi r^2 + \frac{1000}{r}$$

(3)

Given that r can vary,

(b) find the dimensions of a can that has minimum surface area.

(5)

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

(1)

(Total for question = 9 marks)

Q6.

A curve C has equation

$$y = x^2 - 2x - 24\sqrt{x}, \qquad x > 0$$

(a) Find (i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 4$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

(Total for question = 7 marks)

Q7.

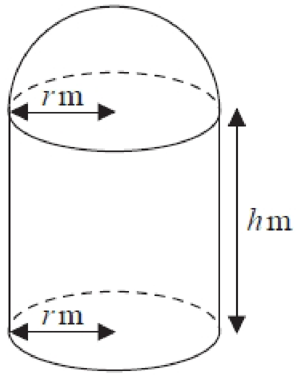

Figure 9

[A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$]

A manufacturer produces a storage tank.

The tank is modelled in the shape of a hollow circular cylinder closed at one end with a hemispherical shell at the other end as shown in Figure 9.

The walls of the tank are assumed to have negligible thickness.

The cylinder has radius r metres and height h metres and the hemisphere has radius r metres.

The volume of the tank is 6 m^3.

(a) Show that, according to the model, the surface area of the tank, in m^2, is given by

$$\frac{12}{r} + \frac{5}{3}\pi r^2$$

(4)

The manufacturer needs to minimise the surface area of the tank.

(b) Use calculus to find the radius of the tank for which the surface area is a minimum.

(4)

(c) Calculate the minimum surface area of the tank, giving your answer to the nearest integer.

(2)

(Total for question = 10 marks)

Q8.

Given that

$$f(x) = x^2 - 4x + 5 \qquad x \in \mathbb{R}$$

(a) express f(x) in the form $(x + a)^2 + b$ where a and b are integers to be found.

(2)

The curve with equation y = f(x)

- meets the y-axis at the point P
- has a minimum turning point at the point Q

(b) Write down

(i) the coordinates of P
(ii) the coordinates of Q

(2)

(Total for question = 4 marks)

Q9.

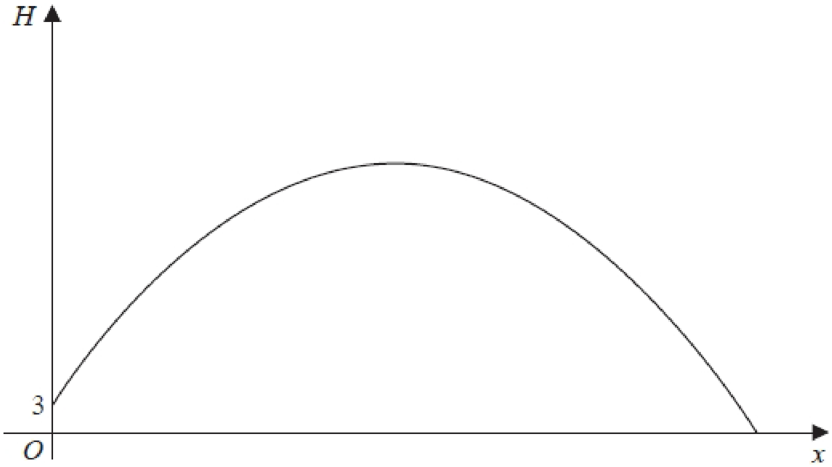

Figure 3

Figure 3 is a graph of the trajectory of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, H metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

Given that the ball

- is hit from a point on the top of a platform of vertical height 3 m above the ground
- reaches its maximum vertical height after travelling a horizontal distance of 90 m
- is at a vertical height of 27 m above the ground after travelling a horizontal distance of 120 m

Given also that H is modelled as a **quadratic** function in x

(a) find H in terms of x

(5)

(b) Hence find, according to the model,

(i) the maximum vertical height of the ball above the ground,
(ii) the horizontal distance travelled by the ball, from when it was hit to when it first hits the ground, giving your answer to the nearest metre.

(3)

(c) The possible effects of wind or air resistance are two limitations of the model.
Give one other limitation of this model.

(1)

(Total for question = 9 marks)

Q10.

The curve C has equation

$y = 5x^4 - 24x^3 + 42x^2 - 32x + 11 \qquad x \in \mathbb{R}$

(a) Find

(i) $\frac{\mathrm{d}y}{\mathrm{d}x}$

(ii) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$

(3)

(b) (i) Verify that C has a stationary point at $x = 1$

(ii) Show that this stationary point is a point of inflection, giving reasons for your answer.

(4)

(Total for question = 7 marks)

Q11.

The curve C has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 2$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

(Total for question = 7 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	Sets $2xy+\frac{\pi x^2}{2}=250$	B1	2.1
	Obtain $y=\frac{250-\frac{\pi x^2}{2}}{2x}$ and substitute into P	M1	1.1b
	Use $P=2x+2y+\pi x$ with their y substituted	M1	2.1
	$P=2x+\frac{250}{x}-\frac{\pi x^2}{2x}+\pi x=2x+\frac{250}{x}+\frac{\pi x}{2}$ *	A1*	1.1b
		(4)	
(b)	$x>0$ and $y>0$ (distance) $\Rightarrow\frac{250-\frac{\pi x^2}{2}}{2x}>0$ or $250-\frac{\pi x^2}{2}>0$ o.e.	M1	2.4
	As x and y are distances they are positive so $0<x<\sqrt{\frac{500}{\pi}}$ *	A1*	3.2a
		(2)	
(c)	Differentiates P with negative index correct in $\frac{dP}{dx}$; $x^{-1}\to x^{-2}$	M1	3.4
	$\frac{dP}{dx}=2-\frac{250}{x^2}+\frac{\pi}{2}$	A1	1.1b
	Sets $\frac{dP}{dx}=0$ and proceeds to $x=$	M1	1.1b
	Substitutes their x into $P=2x+\frac{250}{x}+\frac{\pi x}{2}$ to give perimeter = 59.8m.	A1	1.1b
		(4)	
		(10 marks)	

Notes

(a) B1 : Correct area equation
M1 : Rearranges their area equation to make y the subject of the formula and attempt to use with an expression for P
M1 : Use correct equation for perimeter with their y substituted
A1*: Completely correct solution to obtain and state printed answer
(b) M1 : States $x>0$ and $y>0$ and uses their expression from (a) to form inequality
A1*: Explains that x and y are positive because they are distances, and uses correct expression for y to give the printed answer correctly.
(c) M1: Attempt to differentiate P (deals with negative power of x correctly)
A1 : Correct differentiation
M1 : Sets derived function equal to zero and obtains $x=$
A1: The value of x may not be seen (it is 8.37 to 3sf or $\sqrt{\left(\frac{500}{4+\pi}\right)}$).
Need to see awrt 59.8m with units included for the perimeter.

Q2.

Question	Scheme	Marks	AOs
(a)(i)	$C=\frac{1500}{v}+\frac{2v}{11}+60 \Rightarrow \frac{dC}{dv}=-\frac{1500}{v^2}+\frac{2}{11}$	M1 A1	3.1b 1.1b
	Sets $\frac{dC}{dv}=0 \Rightarrow v^2=8250$	M1	1.1b
	$\Rightarrow v=\sqrt{8250} \Rightarrow v=90.8\ (\text{km h}^{-1})$	A1	1.1b
(ii)	For substituting their $v=90.8$ in $C=\frac{1500}{v}+\frac{2v}{11}+60$	M1	3.4
	Minimum cost =awrt (£) 93	A1 ft	1.1b
		(6)	
(b)	Finds $\frac{d^2C}{dv^2}=+\frac{3000}{v^3}$ at $v=90.8$	M1	1.1b
	$\frac{d^2C}{dv^2}=(+0.004)>0$ hence minimum (cost)	A1 ft	2.4
		(2)	
(c)	It would be impossible to drive at this speed over the whole journey	B1	3.5b
		(1)	
			(9 marks)

Notes

(a)(i)

M1: Attempts to differentiate (deals with the powers of v correctly).

Look for an expression for $\frac{dC}{dv}$ in the form $\frac{A}{v^2}+B$

A1: $\left(\frac{dC}{dv}\right)=-\frac{1500}{v^2}+\frac{2}{11}$

A number of students are solving part (a) numerically or graphically. Allow these students to pick up the M1 A1 here from part (b) when they attempt the second derivative.

M1: Sets $\frac{dC}{dv}=0$ (which may be implied) and proceeds to an equation of the type $v^n=k, k>0$

Allow here equations of the type $\frac{1}{v^n}=k, k>0$

A1: $v=\sqrt{8250}$ or $5\sqrt{330}$ awrt 90.8 (km h^{-1}) .

As this is a speed withhold this mark for answers such as $v=\pm\sqrt{8250}$

* Condone $\frac{dC}{dv}$ appearing as $\frac{dy}{dx}$ or perhaps not appearing at all. Just look for the rhs.

(a)(ii)

M1: For a correct method of finding $C =$ from their solution to $\frac{\mathrm{d}C}{\mathrm{d}v} = 0$.

Do not accept attempts using negative values of v.
Award if you see $v = ..., C = ...$ where the v used is their solution to (a)(i).

A1ft: Minimum cost = awrt (£) 93. Condone the omission of units
Follow through on sensible values of v. $60 < v < 110$

v	C
60	95.9
65	94.9
70	94.2
75	93.6
80	93.3
85	93.1
90	93.0
95	93.1
100	93.2
105	93.4
110	93.6

(b)

M1: Finds $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$ (following through on their $\frac{\mathrm{d}C}{\mathrm{d}v}$ which must be of equivalent difficulty) and attempts to find its value / sign at their v

Allow a substitution of their answer to (a) (i) in their $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$

Allow an explanation into the sign of $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$ from its terms (as $v > 0$)

A1ft: $\frac{\mathrm{d}^2C}{\mathrm{d}v^2} = +0.004 > 0$ hence minimum (cost). Alternatively $\frac{\mathrm{d}^2C}{\mathrm{d}v^2} = +\frac{3000}{v^3} > 0$ as $v > 0$
Requires a correct calculation or expression, a correct statement and a correct conclusion.

Follow through on their v ($v > 0$) and their $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$

* Condone $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$ appearing as $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ or not appearing at all for the M1 but for the A1 the correct notation must be used (accept notation C'').

(c)
B1: Gives a limitation of the given model, for example
- It would be impossible to drive at this speed over the whole journey
- The traffic would mean that you cannot drive at a constant speed

Any statement that implies that the speed could not be constant is acceptable.

Q3.

Question	Scheme	Marks	AOs
(a)	Deduces $g(x) = ax^3 + bx^2 + ax$	B1	2.2a
	Uses $(2, 9) \Rightarrow 9 = 8a + 4b + 2a$ $\Rightarrow 10a + 4b = 9$	M1 A1	2.1 1.1b
	Uses $g'(2) = 0 \Rightarrow 0 = 12a + 4b + a$ $\Rightarrow 13a + 4b = 0$	M1 A1	2.1 1.1b
	Solves simultaneously $\Rightarrow a, b$	dM1	1.1b
	$g(x) = -3x^3 + \frac{39}{4}x^2 - 3x$	A1	1.1b
		(7)	
(b)	Attempts $g''(x) = -18x + \frac{39}{2}$ and substitutes $x = 2$	M1	1.1b
	$g''(2) = -\frac{33}{2} < 0$ hence maximum	A1	2.4
		(2)	
			(9 marks)

Notes

(a)

B1: Uses the information given to deduce that $g(x) = ax^3 + bx^2 + ax$. (Seen or implied)

M1: Uses the fact that $(2, 9)$ lies on the curve so uses $x = 2, y = 9$ within a cubic function

A1: For a simplified equation in just two variables. E.g. $10a + 4b = 9$

M1: Differentiates their cubic to a quadratic and uses the fact that $g'(2) = 0$ to obtain an equation in a and b.

A1: For a different simplified equation in two variables E.g. $13a + 4b = 0$

dM1: Solves simultaneously $\Rightarrow a = ..., b = ...$ It is dependent upon the B and both M's

A1: $g(x) = -3x^3 + \frac{39}{4}x^2 - 3x$

(b)

M1: Attempts $g''(x) = -18x + \frac{39}{2}$ and substitutes $x = 2$. Award for second derivatives of the form $g''(x) = Ax + B$ with $x = 2$ substituted in.
Alternatively attempts to find the value of their $g'(x)$ or $g(x)$ either side of $x = 2$ (by substituting a value for x within 0.5 either side of 2)

A1: $g''(2) = -\frac{33}{2} < 0$ hence maximum. (allow embedded values but they must refer to the sign or that it is less than zero)
If $g'(x) = -9x^2 + \frac{39}{2}x - 3$ or $g(x) = -3x^3 + \frac{39}{4}x^2 - 3x$ is calculated either side of $x = 2$ then the values must be correct or embedded correctly (you will need to check these) they need to compare $g'(x) > 0$ to the left of $x = 2$ and $g'(x) < 0$ to the right of $x = 2$ or $g(x) < 9$ to the left and $g(x) > 9$ to the right of $x = 2$ hence maximum.

Note If they only sketch the cubic function $g(x) = -3x^3 + \frac{39}{4}x^2 - 3x$ then award M1A0

Q4.

	Scheme	Marks	AOs
	The overall method of finding the x coordinate of A.	M1	3.1a
	$y = 2x^3 - 17x^2 + 40x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 6x^2 - 34x + 40$	B1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow 6x^2 - 34x + 40 = 0 \Rightarrow 2(3x-5)(x-4) = 0 \Rightarrow x = ...$	M1	1.1b
	Chooses $x = 4$ $\cancel{x = \frac{5}{3}}$	A1	3.2a
	$\int 2x^3 - 17x^2 + 40x\,\mathrm{d}x = \left[\frac{1}{2}x^4 - \frac{17}{3}x^3 + 20x^2\right]$	B1	1.1b
	Area $= \frac{1}{2}(4)^4 - \frac{17}{3}(4)^3 + 20(4)^2$	M1	1.1b
	$= \frac{256}{3}$ *	A1*	2.1
		(7)	
			(7 marks)

Notes
M1: An overall problem -solving method mark to find the minimum point. To score this you need to see • an attempt to differentiate with at least **two correct terms** • an attempt to set their $\frac{dy}{dx} = 0$ and then solve to find x. Don't be overly concerned by the mechanics of this solution **B1:** $\left(\frac{dy}{dx} =\right) 6x^2 - 34x + 40$ which may be unsimplified **M1:** Sets their $\frac{dy}{dx} = 0$, which must be a 3TQ in x, and attempts to solve via factorisation, formula or calculator. If a calculator is used to find the roots, they must be correct for their quadratic. If $\frac{dy}{dx}$ is correct allow them to just choose the root 4 for M1 A1. Condone $(x-4)\left(x-\frac{5}{3}\right)$ **A1:** Chooses $x = 4$ This may be awarded from the upper limit in their integral **B1:** $\int 2x^3 - 17x^2 + 40x \, dx = \left[\frac{1}{2}x^4 - \frac{17}{3}x^3 + 20x^2\right]$ which may be unsimplified **M1:** Correct attempt at area. There may be slips on the integration but expect **two correct terms** The upper limit used must be their larger solution of $\frac{dy}{dx} = 0$ and the lower limit used must be 0. So if their roots are 6 and 10, then they must use 10 and 0. If only one value is found then the limits must be 0 to that value. Expect to see embedded or calculated values. Don't accept $\int_0^4 2x^3 - 17x^2 + 40x \, dx = \frac{256}{3}$ without seeing the integration and the embedded or calculated values **A1*:** Area $= \frac{256}{3}$ **with** correct notation and no errors. Note that this is a given answer. For correct notation expect to see • $\frac{dy}{dx}$ or $\frac{d}{dx}$ used correctly at least once. If $f(x)$ is used accept $f'(x)$. Condone y' • $\int 2x^3 - 17x^2 + 40x \, dx$ used correctly at least once with or without the limits.

Q5.

Question	Scheme	Marks	AOs
(a)	Sets $500 = \pi r^2 h$	B1	2.1
	Substitute $h = \dfrac{500}{\pi r^2}$ into $S = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r \times \dfrac{500}{\pi r^2}$	M1	2.1
	Simplifies to reach given answer $S = 2\pi r^2 + \dfrac{1000}{r}$ *	A1*	1.1b
		(3)	
(b)	Differentiates S with both indices correct in $\dfrac{dS}{dr}$	M1	3.4
	$\dfrac{dS}{dr} = 4\pi r - \dfrac{1000}{r^2}$	A1	1.1b
	Sets $\dfrac{dS}{dr} = 0$ and proceeds to $r^3 = k, k$ is a constant	M1	2.1
	Radius $= 4.30\text{cm}$	A1	1.1b
	Substitutes their $r = 4.30$ into $h = \dfrac{500}{\pi r^2}$ $\Rightarrow$ Height $= 8.60\text{cm}$	A1	1.1b
		(5)	
(c)	States a valid reason such as • The radius is too big for the size of our hands • If $r = 4.3\text{cm}$ and $h = 8.6\text{cm}$ the can is square in profile. All drinks cans are taller than they are wide • The radius is too big for us to drink from • They have different dimensions to other drinks cans and would be difficult to stack on shelves with other drinks cans	B1	3.2a
		(1)	
		9 marks	

Notes:
(a)
B1: Uses the correct volume formula with V=500. Accept $500 = \pi r^2 h$
M1: Substitutes $h = \dfrac{500}{\pi r^2}$ or $rh = \dfrac{500}{\pi r}$ into $S = 2\pi r^2 + 2\pi r h$ to get S as a function of r
A1*: $S = 2\pi r^2 + \dfrac{1000}{r}$ Note that this is a given answer.
(b)
M1: Differentiates the given S to reach $\dfrac{dS}{dr} = Ar \pm Br^{-2}$
A1: $\dfrac{dS}{dr} = 4\pi r - \dfrac{1000}{r^2}$ or exact equivalent
M1: Sets $\dfrac{dS}{dr} = 0$ and proceeds to $r^3 = k, k$ is a constant
A1: $R =$ awrt 4.30cm
A1: $H =$ awrt 8.60 cm
(c)
B1: Any valid reason. See scheme for alternatives

Q6.

Question	Scheme	Marks	AOs
(a)	(i) $\frac{dy}{dx} = 2x - 2 - 12x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2} = 2 + 6x^{-\frac{3}{2}}$	B1ft	1.1b
		(3)	
(b)	Substitutes $x = 4$ into their $\frac{dy}{dx} = 2\times4 - 2 - 12\times4^{-\frac{1}{2}} = \dots$	M1	1.1b
	Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point" oe	A1	2.1
		(2)	
(c)	Substitutes $x = 4$ into their $\frac{d^2y}{dx^2} = 2 + 6\times4^{-\frac{3}{2}} = (2.75)$	M1	1.1b
	$\frac{d^2y}{dx^2} = 2.75 > 0$ and states "hence minimum"	A1ft	2.2a
		(2)	
			(7 marks)

(a)(i)

M1: Differentiates to $\frac{\mathrm{d}y}{\mathrm{d}x} = Ax + B + Cx^{\frac{1}{2}}$ **A1:** $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x - 2 - 12x^{\frac{1}{2}}$ (Coefficients may be unsimplified)

(a)(ii)

B1ft: Achieves a correct $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ for their $\frac{\mathrm{d}y}{\mathrm{d}x}$ (Their $\frac{\mathrm{d}y}{\mathrm{d}x}$ must have a negative or fractional index)

(b)

M1: Substitutes $x = 4$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and attempts to evaluate. There must be evidence $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=4} = \ldots$

Alternatively substitutes $x = 4$ into an equation resulting from $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ Eg. $\frac{36}{x} = (x-1)^2$ and equates

A1: There must be a reason and a minimal conclusion. Allow ✓ , QED for a minimal conclusion

Shows $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and states "hence there is a stationary point" oe

Alt Shows that $x = 4$ is a root of the resulting equation and states "hence there is a stationary point"

All aspects of the proof must be correct including a conclusion

(c)

M1: Substitutes $x = 4$ into their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and calculates its value, or implies its sign by a statement such as when $x = 4 \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0$. This must be seen in (c) and not labelled (b). Alternatively calculates the gradient of C either side of $x = 4$ or calculates the value of y either side of $x = 4$.

A1ft: For **a correct calculation**, **a valid reason** and **a correct conclusion**. Ignore additional work where candidate finds $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ left and right of $x = 4$. Follow through on an incorrect $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ but it is dependent upon having a negative or fractional index. Ignore any references to the word convex. The nature of the turning point is "minimum".

Using the gradient look for correct calculations, a valid reason…. goes from negative to positive, and a correct conclusion …minimum.

Q7.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$6 = \pi r^2 h + \frac{2}{3}\pi r^3$	B1	This mark is given for a method to find the volume of the cylinder and the semi-hemisphere
	$A = 3\pi r^2 + 2\pi\left(\frac{6 - \frac{2}{3}\pi r^3}{\pi r}\right)$	M1	This mark is given for a method to find the surface area of the tank
		A1	This mark is given for finding an expression for the surface area of the tank
	$A = 3\pi r^2 + \frac{12}{r} - \frac{4\pi r^2}{3} = \frac{12}{r} + \frac{5\pi r^2}{3}$	A1	This mark is given for a fully correct proof to show the surface area of the tank as required
(b)	$A = \frac{12}{r} + \frac{5\pi r^2}{3} \Rightarrow \frac{dA}{dr} = -\frac{12}{r^2} + \frac{10\pi r}{3}$	M1	This mark is given for a method to differentiate to find r
		A1	This mark is given for accurately differentiating to find r
	When $\frac{dA}{dr} = 0$, $-\frac{12}{r^2} + \frac{10\pi r}{3} = 0$ $r^3 = \frac{18}{5\pi}$	M1	This mark is given for a method to set $\frac{dA}{dr} = 0$ to find a value for r
	$r = 1.046$	A1	This mark is given for finding the radius for which the surface area is a minimum
(c)	$A = \frac{12}{1.046} + \frac{5\pi(1.046)^2}{3}$	M1	This mark is given for a method to substitute a value for r
	$A = 17\text{ m}^2$	A1	This mark is given for correctly finding the minimum surface area of the tank (to the nearest integer)
			(Total 10 marks)

Q8.

Question	Scheme	Marks	AOs
(a)	$\mathrm{f}(x)=(x-2)^2 \pm ...$	M1	1.2
	$\mathrm{f}(x)=(x-2)^2+1$	A1	1.1b
		(2)	
(b)(i)	$P=(0, 5)$	B1	1.1b
(b)(ii)	$Q=(2, 1)$	B1ft	1.1b
		(2)	
			(4 marks)
Notes			

(a)

M1: Achieves $(x-2)^2 \pm ...$ or states $a=-2$

A1: Correct expression $(x-2)^2+1$ ISW after sight of this

Condone $a=-2$ and $b=1$. Condone $(x-2)^2+1=0$

(b)

(i) B1: Correct coordinates for P. Allow to be expressed $x=0, y=5$

(ii) B1ft: Correct coordinates for Q. Allow to be expressed $x=2, y=1$ (Score for the correct answer or follow through their part (a) so allow $(-a, b)$ where a and b are numeric)

Score in any order if they state $P=(0, 5)$ and $Q=(2, 1)$

Allow part (b) to be awarded from a sketch. So award

First B1 from a sketch crossing the y-axis at 5

Second B1 from a sketch with minimum at $(2, 1)$

Q9.

Question	Scheme	Marks	AOs
(a)	$H = ax^2 + bx + c$ and $x=0,\ H=3 \Rightarrow H = ax^2+bx+3$	M1	3.3
	$H = ax^2 + bx + 3$ and $x=120, H=27 \Rightarrow 27 = 14400a + 120b + 3$ or $\frac{dH}{dx} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$	M1 A1	3.1b 1.1b
	$H = ax^2 + bx + 3$ and $x=120, H=27 \Rightarrow 27 = 14400a + 120b + 3$ **and** $\frac{dH}{dx} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$ $\Rightarrow a = ..., b = ...$	dM1	3.1b
	$H = -\frac{1}{300}x^2 + \frac{3}{5}x + 3$ o.e.	A1	1.1b
		(5)	
(b)(i)	$x = 90 \Rightarrow H\left(= -\frac{1}{300}(90)^2 + \frac{3}{5}(90) + 3\right) = 30\,\text{m}$	B1	3.4
(b)(ii)	$H = 0 \Rightarrow -\frac{1}{300}x^2 + \frac{3}{5}x + 3 = 0 \Rightarrow x = ...$	M1	3.4
	$x = (-4.868...,)\ 184.868...$ $\Rightarrow x = 185\,(\text{m})$	A1	3.2a
		(3)	
(c)	Examples must focus on why the model may not be appropriate or give values/situations where the model would break down: E.g. • The ground is unlikely to be horizontal • The ball is not a particle so has dimensions/size • The ball is unlikely to travel in a vertical plane (as it will spin) • H is not likely to be a quadratic function in x	B1	3.5b
		(1)	
			(9 marks)
	Notes		

....

(a)

M1: Translates the problem into a suitable model and uses $H = 3$ when $x = 0$ to establish $c = 3$

Condone with $a = \pm 1$ so $H = x^2 + bx + 3$ will score M1 but little else

M1: For a correct attempt at **using one of the two other pieces** of information within a quadratic model **Either** uses $H = 27$ when $x = 120$ (with $c = 3$) to produce a linear equation connecting a and b for the model **Or** differentiates and uses $\frac{dH}{dx} = 0$ when $x = 90$. Alternatives exist here, using the symmetrical nature of the curve, so they could use $x = -\frac{b}{2a}$ at vertex or use point (60, 27) or (180,3).

A1: At least one correct equation connecting a and b. Remember "a" could have been set as negative so an equation such as $27 = -14400a + 120b + 3$ would be correct in these circumstances.

dM1: Fully correct strategy that uses $H = ax^2 + bx + 3$ with the two other pieces of information in order to establish the values of **both** a **and** b for the model

A1: Correct equation, not just the correct values of a, b and c. Award if seen in part (b)

(b)(i)

B1: Correct height including the units. CAO

(b)(ii)

M1: Uses $H = 0$ and attempts to solve for x. Usual rules for quadratics.

A1: Discards the negative solution (may not be seen) and identifies awrt 185 m. Condone lack of units

(c)

B1: Candidate should either refer to an issue with one of the four aspects of how the situation has been modelled or give a situation where the model breaks down

- the ball has been modelled as a particle
- there may be trees (or other hazards) in the way that would affect the motion

Condone answers (where the link to the model is not completely made) such as

- the ball will spin
- ground is not flat

Do not accept answers which refer to the situation after it hits the ground (this isn't what was modelled)

- the ball will bounce after hitting the ground
- it gives a negative height for some values for x

Do not accept answers that do not refer to the model in question, or else give single word vague answers

- the height of tee may have been measured incorrectly
- "friction", "spin", "force" etc
- it does not take into account the weight of the ball
- it depends on how good the golfer is
- the shape of the ball will affect the motion
- you cannot hit a ball the same distance each time you hit it

The method using an alternative form of the equation can be scored in a very similar way.
The first M is for the completed square form of the quadratic showing a maximum at $x = 90$

So award M1 for $H = \pm a(x-90)^2 + c$ or $H = \pm a(90-x)^2 + c$. Condone for this mark an equation with $a = 1 \Rightarrow H = (x-90)^2 + c$ or $c = 3 \Rightarrow H = a(x-90)^2 + 3$ but will score little else

Alt (a)	$H = a(x+b)^2 + c$ and $x = 90$ at $H_{max} \Rightarrow H = a(x-90)^2 + c$	M1	3.3
	$H = 3$ when $x = 0 \Rightarrow 3 = 8100a + c$ **or** $H = 27$ when $x = 120 \Rightarrow 27 = 900a + c$	M1 A1	3.1b 1.1b
	$H = 3$ when $x = 0 \Rightarrow 3 = 8100a + c$ **and** $H = 27$ when $x = 120 \Rightarrow 27 = 900a + c$ $\Rightarrow a = ..., c = ...$	dM1	3.1b
	$H = -\dfrac{1}{300}(x-90)^2 + 30$ o.e	A1	1.1b
		(5)	
(b)	$x = 90 \Rightarrow H = 0^2 + 30 = 30\text{m}$	B1	3.4
		(1)	
	$H = 0 \Rightarrow 0 = -\dfrac{1}{300}(x-90)^2 + 30 \Rightarrow x = ...$	M1	3.4
	$\Rightarrow x = 185\,(\text{m})$	A1	3.2a
		(2)	

Note that $H=-\frac{1}{300}(x-90)^2+30$ is equivalent to $H=-\frac{1}{300}(90-x)^2+30$

Other versions using symmetry are also correct so please look carefully at all responses

E.g. Using a starting equation of $H=a(x-60)(x-120)+b$ leads to $H=-\frac{1}{300}(x-60)(x-120)+27$

Q10.

Question	Scheme	Marks	AOs
(a)(i)	$\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$	M1 A1	1.1b 1.1b
(ii)	$\frac{d^2y}{dx^2} = 60x^2 - 144x + 84$	A1ft	1.1b
		(3)	

(b)(i)	$x = 1 \Rightarrow \frac{dy}{dx} = 20 - 72 + 84 - 32$	M1	1.1b
	$\frac{dy}{dx} = 0$ so there is a stationary point at $x = 1$	A1	2.1
	Alternative for (b)(i)		
	$20x^3 - 72x^2 + 84x - 32 = 4(x-1)^2(5x-8) = 0 \Rightarrow x = ...$	M1	1.1b
	When $x = 1$, $\frac{dy}{dx} = 0$ so there is a stationary point	A1	2.1
(b)(ii)	**Note that in (b)(ii) there are no marks for just evaluating** $\left(\frac{d^2y}{dx^2}\right)_{x=1}$		
	E.g. $\left(\frac{d^2y}{dx^2}\right)_{x=0.8} = ...$ $\left(\frac{d^2y}{dx^2}\right)_{x=1.2} = ...$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=0.8} > 0$, $\left(\frac{d^2y}{dx^2}\right)_{x=1.2} < 0$ Hence point of inflection	A1	2.2a
		(4)	
	Alternative 1 for (b)(ii)		
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 60x^2 - 144x + 84 = 0$ (is inconclusive) $\left(\frac{d^3y}{dx^3}\right) = 120x - 144 \Rightarrow \left(\frac{d^3y}{dx^3}\right)_{x=1} = ...$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 0$ and $\left(\frac{d^3y}{dx^3}\right)_{x=1} \neq 0$ Hence point of inflection	A1	2.2a
	Alternative 2 for (b)(ii)		
	E.g. $\left(\frac{dy}{dx}\right)_{x=0.8} = ...$ $\left(\frac{dy}{dx}\right)_{x=1.2} = ...$	M1	2.1
	$\left(\frac{dy}{dx}\right)_{x=0.8} < 0$, $\left(\frac{dy}{dx}\right)_{x=1.2} < 0$ Hence point of inflection	A1	2.2a
			(7 marks)

(a)(i)
M1: $x^n \to x^{n-1}$ for at least one power of x
A1: $\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$
(a)(ii)
A1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$
(b)(i)
M1: Substitutes $x = 1$ into their $\frac{dy}{dx}$
A1: Obtains $\frac{dy}{dx} = 0$ following a correct derivative and makes a conclusion which can be minimal e.g. tick, QED etc. which may be in a preamble e.g. stationary point when $\frac{dy}{dx} = 0$ and then shows $\frac{dy}{dx} = 0$
Alternative:
M1: Attempts to solve $\frac{dy}{dx} = 0$ by factorisation. This may be by using the factor of $(x - 1)$ or possibly using a calculator to find the roots and showing the factorisation. Note that they may divide by 4 before factorising which is acceptable. Need to either see either $4(x-1)^2(5x-8)$ or $(x-1)^2(5x-8)$ for the factorisation or $x = \frac{8}{5}$ and $x = 1$ seen as the roots.
A1: Obtains $x = 1$ and makes a conclusion as above

(b)(ii)

M1: Considers the value of the second derivative either side of $x = 1$. Do not be too concerned with the interval for the method mark.

(NB $\frac{d^2y}{dx^2} = (x-1)(60x-84)$ so may use this factorised form when considering $x < 1$, $x > 1$ for sign change of second derivative)

A1: Fully correct work including a correct $\frac{d^2y}{dx^2}$ with a reasoned conclusion indicating that the stationary point is a point of inflection. Sufficient reason is e.g. "sign change"/ "> 0, < 0". If values are given they should be correct (but be generous with accuracy) but also just allow "> 0" and "< 0" provided they are correctly paired. The interval must be where $x < 1.4$

Alternative 1 for (b)(ii)

M1: Shows that second derivative at $x = 1$ is zero and **then finds the third derivative at $x = 1$**

A1: Fully correct work including a correct $\frac{d^2y}{dx^2}$ with a reasoned conclusion indicating that stationary point is a point of inflection. Sufficient reason is "≠ 0" but must follow a correct third derivative and a correct value if evaluated. For reference $\left(\frac{d^3y}{dx^3}\right)_{x=1} = -24$

Alternative 2 for (b)(ii)

M1: Considers the value of the first derivative either side of $x = 1$. Do not be too concerned with the interval for the method mark.

A1: Fully correct work with a reasoned conclusion indicating that stationary point is a point of inflection. Sufficient reason is e.g. "same sign"/"both negative"/"< 0, < 0". If values are given they should be correct (but be generous with accuracy). The interval must be where $x < 1.4$

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
f'(x)	-32	-24.3	-17.92	-12.74	-8.64	-5.5	-3.2	-1.62	-0.64	-0.14	0
f''(x)	84	70.2	57.6	46.2	36	27	19.2	12.6	7.2	3	0

x	1.1	1.2	1.3	1.4	1.5	1.6	1.7
f'(x)	-0.1	-0.32	-0.54	-0.64	-0.5	0	0.98
f''(x)	-1.8	-2.4	-1.8	0	3	7.2	12.6

Q11.

Question	Scheme	Marks	AOs
(a)	(i) $\frac{dy}{dx} = 12x^3 - 24x^2$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2} = 36x^2 - 48x$	A1ft	1.1b
		(3)	
(b)	Substitutes $x = 2$ into their $\frac{dy}{dx} = 12\times2^3 - 24\times2^2$	M1	1.1b
	Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point"	A1	2.1
		(2)	
(c)	Substitutes $x = 2$ into their $\frac{d^2y}{dx^2} = 36\times2^2 - 48\times2$	M1	1.1b
	$\frac{d^2y}{dx^2} = 48 > 0$ and states "hence the stationary point is a minimum"	A1ft	2.2a
		(2)	
		(7 marks)	

Notes:

(a)(i)

M1: Differentiates to a cubic form

A1: $\frac{dy}{dx} = 12x^3 - 24x^2$

(a)(ii)

A1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx} = 36x^2 - 48x$

(b)

M1: Substitutes $x = 2$ into their $\frac{dy}{dx}$

A1: Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point" All aspects of the proof must be correct

(c)

M1: Substitutes $x = 2$ into their $\frac{d^2y}{dx^2}$

Alternatively calculates the gradient of C either side of $x = 2$

A1ft: For a correct calculation, a valid reason and a correct conclusion.

Follow through on an incorrect $\frac{d^2y}{dx^2}$